Mantle cloak: Invisibility induced by a surface

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(Received 29 October 2009; revised manuscript received 30 November 2009; published 21 December 2009)

The exotic wave interaction of metamaterials has been recently applied to various cloaking applications, but the metamaterial realization in practical cloaks is still far from ideal. Current fabrication techniques are inherently based on the volumetric properties of metamaterials, which require a non-negligible electrical thickness. I introduce here the idea of surface cloaking, showing that a patterned metasurface may produce similar cloaking effects in a simpler and thinner geometry. The currents induced on a passive surface are tailored to drastically suppress the visibility of a given object.

DOI: 10.1103/PhysRevB.80.245115 PACS number(s): 42.70.−a, 33.20.Fb, 42.50.Gy, 42.79.−e

I. INTRODUCTION

Recent findings in metamaterial technology have shown that invisibility, transparency and cloaking may be obtained with various techniques based on the complex wave interaction of artificial materials and metamaterials (see, e.g., Refs. 1 and 2). Transformation-based cloaking[3–6] is currently one of the most popular techniques, and practical attempts to extend its experimental realization to visible frequencies have been recently made.[7] The operation of these cloaks is based on the electromagnetic properties of bulk metamaterials with specific anisotropy and inhomogeneity profiles, which may bend electromagnetic waves around a given region of space, isolating it electromagnetically and making invisible any object placed inside such region. As other viable techniques, plasmonic cloaking[8,9] is based on the scattering cancellation features of low-permittivity metamaterials that may get polarized in unusual ways, whereas anomalous localized resonant mechanisms[10] are based on the quasistatic resonant properties of metamaterials that may effectively cloak a given region of interest. All these techniques, as well as various others that involve metamaterial cloaks, are based on the specific *bulk volumetric properties* of metamaterial layers. More in general, these artificial materials are based on the *collective* electromagnetic response of their constituent inclusions, which interact as a bulk with the impinging electromagnetic wave, obtaining effects very distinct from those of the individual materials they are composed of. If this may provide large degrees of freedom to produce anomalous effects, such as cloaking, on the other hand metamaterial cloaks inherently require a certain thickness, due to the finite size of their constituent inclusions. In the case of transformation-based cloaks, in particular, the involved inhomogeneity profile requires cloaks that have a thickness that is comparable in size with the region to be cloaked. Moreover, some additional spacing is usually required between the metamaterial cloak and the object to be cloaked, in order to ensure that the metamaterial granularity does not produce unwanted coupling with the object, which may affect its overall electromagnetic properties.[11] Having a thicker cloak is not only impractical and undesirable but it also implies reduced bandwidth and increased sensitivity.[12] Even the plasmonic cloaking technique, which requires relatively thinner cloaks than transformation-based metamaterials, may require practically a finite thickness for proper operation.[11,13]

In a different field, the concept of patterned thin metallic surfaces is well established in various engineering applications, with relevant books and reviews on the topic (see, e.g., Ref. 14). Provided that the periodic pattern on a metallic surface is much smaller than the wavelength of operation, its electromagnetic behavior may be effectively described by an averaged surface impedance $Z_s = R_s - iX_s$, which relates the averaged tangential electric field on the surface to the averaged induced electric current density as $\mathbf{E}_{\tan} = Z_s \mathbf{J}$. The impedance Z_s can usually assume a wide range of values as a function of geometry and frequency, from which the name of "frequency selective surface" (FSS). In the lossless scenario, it is easy to prove that Z_s is purely imaginary, R_s being only related to absorption. In the more general scenario, however, the value of Z_s may depend on the orientation of the tangential electric field, implying anisotropy and a tensorial form of $\underline{\mathbf{Z}}_s$. The scalar notation may be still applicable for specific polarizations of the impinging wave.

In the following, I show that a single patterned FSS may be sufficient to produce a cloaking effect analogous to the one of metamaterial cloaks, even in the ideal limit of a surface with zero thickness. This may produce thin cloaks in an established technology, with a long history of applications, the promise of easier realization, possible conformability to the shape of the object, low profile and relatively larger bandwidths of operation. Similar concepts may also be extended to metallic surfaces at tetrahertz and optical frequencies, opening promising venues for the realization of thin optical cloaks with enhanced performance. A *mantle cloak*, as the one suggested here, may make us closer to the practical realization of cloaking, since it would not rely on the bulk constitutive properties of a composite material, but just on the transverse impedance of a patterned metallic surface. It should be mentioned that hard and soft FSSs have been applied in the past to scattering reduction devices,[15] but using concepts drastically different from what considered here. Here, the interest resides in realizing a cloaking mechanism that is independent of the angle of incidence and possibly on the wave polarization, as discussed in the following.

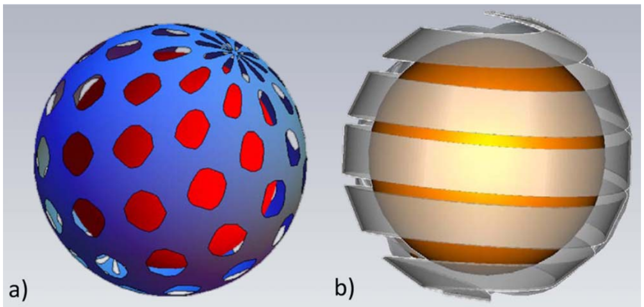

FIG. 1. (Color online) Examples of patterned metallic geometries that may realize a mantle cloak.

II. THEORETICAL FORMULATION

Consider the sample geometries in Fig. 1, i.e., dielectric spheres of radius a covered by a patterned metallic spherical surface with negligible thickness and radius $a_c > a$. These represent typical patterns that may be realized on a metallic surface to provide a quasihomogeneous surface reactance of desired value. Provided that the patterns are subwavelength, they may provide a quasihomogeneous surface reactance of desired value.

One extreme limit arises when the conducting surface has no holes, which would provide an effectively zero tangential electric field, yielding $X_s=0$. In the other extreme scenario of no metal at all, the effective surface reactance would be $X_s \to \infty$. As shown in Ref. 14 and references therein, proper choice of the patterns on the metallic surface allows achieving desired positive or negative values of X_s at the frequency of interest. After homogenization, the scattering problem for given arbitrary excitation may be analytically solved by forcing the required discontinuity of the tangential magnetic field on the cloak surface, at $r=a_c$, proportional to the averaged current induced on the surface. This implies that the boundary condition

$$\mathbf{H}_{\tan}|_{r=a_c^+} - \mathbf{H}_{\tan}|_{r=a_c^-} = \hat{\mathbf{r}} \times \mathbf{E}_{\tan}|_{r=a_c} / Z_s \tag{1}$$

holds in the isotropic case. The Mie scattering solution of this problem implies that the nth TM spherical scattering harmonic may be suppressed, provided that the following determinant is canceled:

$$\begin{vmatrix} j_n(ka) & j_n(k_0a) & y_n(k_0a) & 0 \\ [kaj_n(ka)]'/\varepsilon & [k_0aj_n(k_0a)]' & [k_0ay_n(k_0a)]' & 0 \\ 0 & j_n(k_0a_c)+[k_0a_cj_n(k_0a_c)]'/(i\omega\varepsilon_0a_cZ_s) & y_n(k_0a_c)+[k_0a_cy_n(k_0a_c)]'/(i\omega\varepsilon_0a_cZ_s) & j_n(k_0a_c) \\ 0 & [k_0a_cj_n(k_0a_c)]' & [k_0a_cy_n(k_0a_c)]' & [k_0a_cj_n(k_0a_c)]' \end{vmatrix}, \tag{2}$$

where $j_n(\cdot)$ and $y_n(\cdot)$ are spherical Bessel functions, k and k_0 are the wave numbers in the object and in free space, respectively, ε is the relative permittivity of the object, and ε_0 is the free-space permittivity. For TE spherical harmonics, the dual of Eq. (2) may be easily derived. Provided that the determinant in Eq. (2) may be made very close to zero for the dominant scattering orders of interest, the visibility of a given object would be drastically reduced, independent of the polarization and form of excitation and the position of the observer, achieving a substantial, although not ideal due to the residual scattering orders, cloaking effect. Extension to anisotropic surfaces and a tensorial $\underline{\mathbf{Z}}_s$ may also be envisioned, producing in general a cross coupling between the two polarizations. It is noticed, however, that the isotropic formulation [Eq. (2)] is applicable even for an anisotropic surface for specific polarizations of the impinging wave.

It is instructive to analyze Eq. (2) in the quasistatic limit (electrically small objects), for which $(k_0a_c) \ll 1$. In this case, the dominant contribution to scattering is given by the $n=1$ dominant harmonic and the approximate conditions for cloaking in the two polarizations may be written in an explicit form as

$$\mathrm{TM}: X_s = \frac{2[2+\varepsilon-\gamma^3(\varepsilon-1)]}{3\gamma^3\omega a\varepsilon_0(\varepsilon-1)},$$

$$\mathrm{TE}: X_s = \frac{\omega a\mu_0[2+\mu+2\gamma^3(\mu-1)]}{6\gamma^3(\mu-1)}, \tag{3}$$

where μ refers to permeability and $\gamma=a/a_c$.

Equations (3) show that in the quasistatic limit TE and TM scattering contributions are decoupled, as expected, and it ensures that the dominant multipolar terms, without preference either TM (electric) or TE (magnetic), may both be suppressed by the proper choice of FSS reactance. Despite being constituted by just a thin surface conformal to the object, zero scattering may be achieved in this limit with a lossless reactive surface, and even considering the presence of realistic losses in the metal, the cloaking effect would not be sensibly affected. When the size of the object increases, dynamic formulas as in Eq. (2) may be used for the correct design of a mantle cloak. This formulation may be extended to the case of conducting objects and different nonsymmetrical geometries and anisotropies, without sensibly affecting these results.

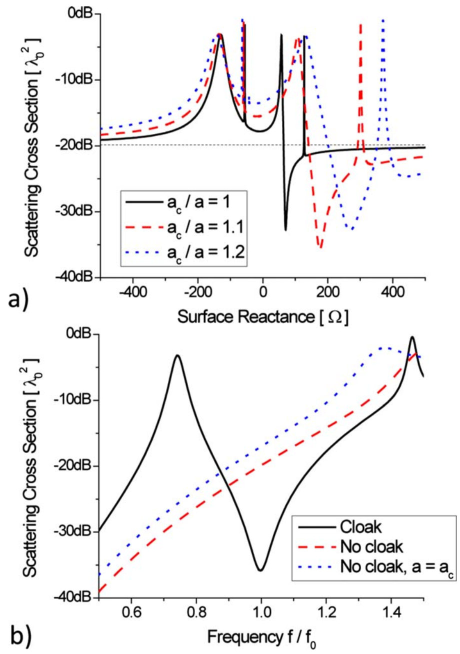

FIG. 2. (Color online) Variation in the total scattering cross-section of a dielectric sphere with $\varepsilon=10$ and $2a=\lambda_0/5$ with: (a) the surface reactance of a mantle cloak; (b) the normalized frequency of operation for a cloak with $a_c=1.1a$ and $X_s=175\ \Omega$.

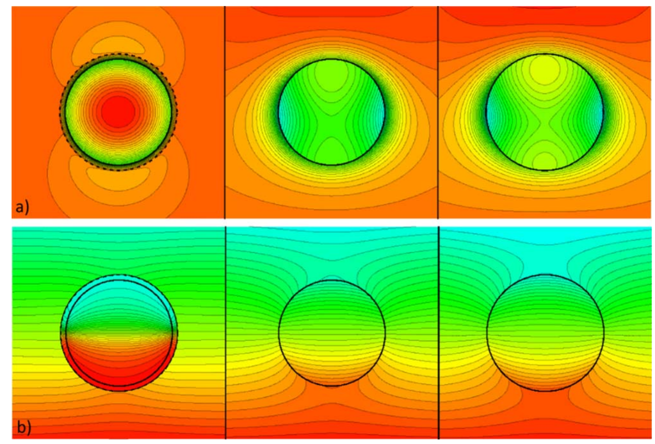

FIG. 3. (Color online) Distributions of the: (a) amplitude of electric field on the H plane; (b) phase of magnetic field on the E plane, for the three scenarios of: a cloaked sphere (left), the same sphere, but uncloaked (center), an enlarged uncloaked dielectric sphere with $a=a_c$ (right). Brighter colors correspond to higher values of the fields. The panels are all in the same scale for a fair comparison.

III. NUMERICAL RESULTS

Consider, as an example, a dielectric sphere with permittivity $\varepsilon=10$ and diameter $2a=\lambda_0/5$, with λ_0 the free-space wavelength. Figure 2(a) reports the variation in the total scattering cross-section varying the surface reactance X_s of the mantle cloak, as compared to the bare sphere (thin dashed line). It is evident that for sufficiently large reactance values the patterned surface does not have any effect on the scattering (the limit of no surface is given by $X_s \to \infty$), but for specific inductive values, qualitatively consistent with Eq. (3) (although with some deviation due to the relatively large electrical size of the object), a relevant scattering reduction is achieved. This may be obtained for different values of a_c, even in the limit of a surface conformal to the object ($\gamma=1$). It is noticed that for other values of reactance, strong resonances for different scattering orders may also be obtained, which, although not relevant here, may be fully characterized within this analysis. For the case of $a_c=1.1a$, over 97% of scattering reduction may be achieved with a surface reactance of 175 Ω. This is obtained without the need of any bulk metamaterial layer, but just using the impedance properties of a properly tailored metallic surface acting as a mantle cloak.

It is noticed in Fig. 2(a) that the surface cloak may provide good performance over a relatively broad range of reactances, being based on a scattering cancellation mechanism in some senses analogous to the plasmonic cloaking technique,[8] although not based on the negative polarization of the metamaterial cover, but rather on specific current patterns induced on the FSS and fields excited between the surface and the object. This explains the robustness and bandwidth of this cloaking mechanism, despite the extremely low profile of the cloak.[13] Figure 2(b) reports the frequency response of the cloak of Fig. 2(a) with $a_c=1.1a$ and $X_s=175\ \Omega$, where it is assumed that the surface reactance is constant with frequency. This assumption may be considered valid over a range of frequencies around the design frequency f_0. It is noticed that a significant scattering reduction may be achieved over a large range of frequencies, when compared to the uncloaked scenario (dashed red line) or to an uncloaked particle with $a=a_c$ (dotted blue). Clearly, the response of the FSS may not be considered completely nondispersive over a wide frequency range, but it is noticed that, due to the absence of volumetric fields, the constraints on its dispersion are more relaxed than the typical dispersion models for metamaterials. This reflects in a surface reactance that is weakly dispersive with frequency, and predictably larger bandwidths of operation, consistent with our full-wave results reported in the following.

Figure 3 reports the amplitude of the electric field distribution on the H plane and the phase of the magnetic field distribution on the E plane for the three scenarios of: the cloaked sphere (left column); the bare sphere (center); and a bare sphere with $a=a_c$ (right) for the geometry of Fig. 2(b) at frequency f_0. We have assumed plane-wave excitation impinging from the bottom of each panel. The different panels are plotted in the same color scale, for a fair comparison. It is visible that the mantle cloak indeed produces a strong scattering reduction, thanks to the proper choice of its reactive surface impedance, restoring almost uniform amplitude and planar phase fronts all around the cloak in both planes of polarization, consistent with the 97% scattering reduction predicted in Fig. 2. Interestingly, it is visible how the field may penetrate the surface and propagate inside the region between the cloak and the sphere, and in the sphere itself, ensuring better robustness and bandwidths[12] compared to cloaking techniques that bend the wave around the object. This may also provide the intriguing possibility to sense and extract the signal inside the surface without significantly per-

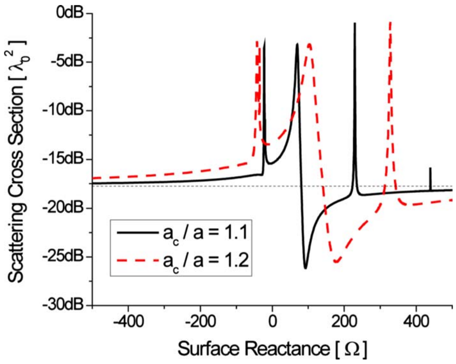

FIG. 4. (Color online) Analogous to Fig. 2(a), but for a conducting sphere of same size.

turbing the surrounding electromagnetic field.[16] This mantle cloak may therefore also represent a viable way for noninvasive sensing and probing[17] with improved bandwidth. The field distributions of Fig. 3 may also highlight the difference between the physical mechanisms at the basis of this cloak compared with the plasmonic cloaking technique.[8] Here, enhanced magnetic field is induced near the surface to sustain the patterned currents, whereas the electric field is maximized at the center of the object. This is drastically different from the plasmonic cloaking technique, for which the magnetic field is continuous at the cloak surface, whereas the electric field is usually maximized near the cloak rather than inside the dielectric object. This may have further advantages in terms of the probing applications of this cloaking technique.

Figures 4 and 5 report similar simulations as in Figs. 2(a) and 3, but for the case of a conducting (impenetrable) sphere with same size. It is evident that in this scenario the wave cannot penetrate the object, and therefore nonzero spacing is required between mantle cloak and object. This is clear, since in the case of $a=a_c$ the surface reactance would be shorted by the metallic surface of the object. However, even a narrow spacing between the cloak and the object may ensure proper cloaking, as shown in Fig. 4. As an aside, this geometry is particularly interesting since it may be used to realize an FSS cloak independent of the object to be cloaked (by

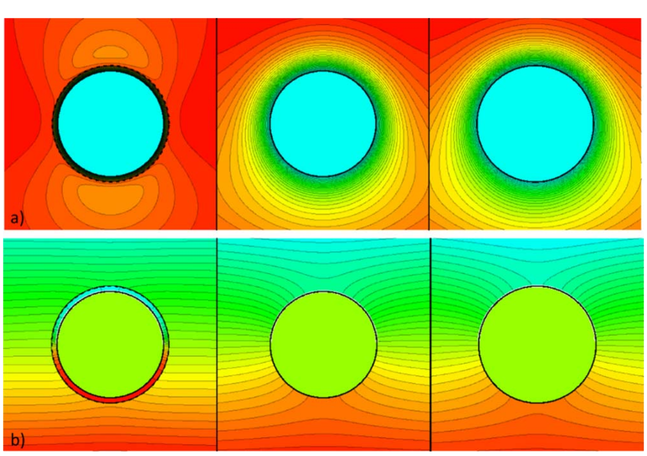

FIG. 5. (Color online) Analogous to Fig. 3, but for a conducting sphere of same size.

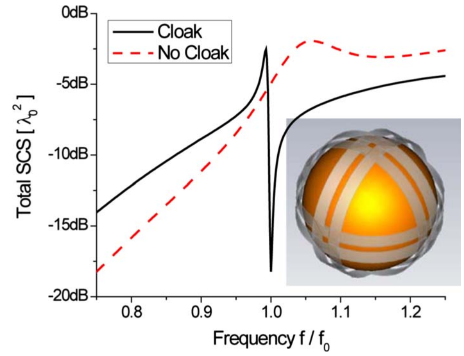

FIG. 6. (Color online) Full-wave simulation results for a realistic quasi-isotropic inductive FSS cloak covering the dielectric sphere of Fig. 2.

placing the cloaked object inside the spherical conducting cavity). As seen in Fig. 4, the price to be paid is reduced bandwidth and slightly worsened cloaking performance. The field plots in Fig. 5 refer to the case of $a_c=1.1a$, with optimized reactance, as derived in Eq. (2), $X_s=92\ \Omega$. The overall scattering reduction in this case is around 86%, a little less than in the dielectric scenario due to stronger scattering contribution from higher-order modes. Still, despite the impenetrability of the object, a simple and thin reactive surface is able to provide drastic reduction in visibility in all directions and in both planes of polarization.

Finally, in Fig. 6 we report the full-wave simulation of a realistic quasi-isotropic FSS realized to cloak the object of Fig. 2 (the surface geometry is reported in the inset of Fig. 6). The cloak is constituted by six orthogonal conducting interconnected stripes that may ensure a quasi-isotropic inductive response around the frequency of interest f_0, for which the object is about $\lambda_0/3$ in size. The strips may provide the required averaged isotropic inductance in the three planes of polarization. Notice the relatively broad bandwidth of operation of the mantle cloak, consistent with the previous considerations on the possibility to realize a weakly dispersive surface reactance. Moreover, this geometry is quasi-isotropic, since its response is weakly affected by changes in polarization or angle of incidence, as evident from its design. The simplicity of this cloak design makes it particularly appealing for moderately sized object. Extensions to electrically larger objects would require the suppression of several scattering orders, which may require the use of multiple stacked FSS or more complicated scenarios. We have verified that for dielectric objects up to a couple of electrical wavelengths large, significant scattering reduction may be achieved with a single FSS layer as the one reported here.

IV. CONCLUSIONS

I have introduced here the idea of cloaking with low-profile thin patterned surfaces, showing that the currents and fields induced on the FSS may produce cloaking for dielectric and conducting objects in a simple and practical geometry over a relatively broad range of frequencies. The theoretical analysis is supported by numerical results and by the design of a realistic isotropic FSS cloak for a dielectric sphere. Similar concepts may also be translated to tetrahertz and optical frequencies, using patterned plasmonic surfaces. In this case, better results are predicted for surfaces thicker than the metal skin depth, for which a similar analysis as in this paper would apply. Even for thinner surfaces realistic cloaking effects may be envisioned at optical frequencies, properly defining the surface impedance as a function of both conducting and displacement currents induced on its surface.[18] This venue may become very promising for the realization of optical cloaks applied to moderately sized objects, such as nanoparticles and microscope tips.[16,17] It is also envisioned that these concepts may be extended to cascaded reactive surfaces, which may provide multifrequency operation and/or cancel multiple scattering orders in the case of larger objects. This would be consistent with,[9] but now employing thin patterned surfaces with proper reactive response. These venues will be explored in the near future.

ACKNOWLEDGMENT

The author is grateful to Pai-Yen Chen for his support in the numerical simulation and design reported in Fig. 6.

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